# Limits on Active-Sterile Neutrino Mixing and the Primordial Deuterium Abundance

Christian Y. Cardall and George M. Fuller

*Department of Physics, University of California, San Diego, La Jolla, California 92093-0319*

(February 1, 2008)



## Abstract

Studies of limits on active-sterile neutrino mixing derived from big bang nucleosynthesis considerations are extended to consider the dependance of these constraints on the primordial deuterium abundance. This study is motivated by recent measurements of D/H in quasar absorption systems, which at present yield discordant results. Limits on active-sterile mixing are somewhat relaxed for high D/H. For low D/H ($\approx 2 \times 10^{-5}$), no active-sterile neutrino mixing is allowed by currently popular upper limits on the primordial $^4$He abundance $Y$. For such low primordial D/H values, the observational inference of active-sterile neutrino mixing by upcoming solar neutrino experiments would imply that $Y$ has been systematically underestimated, unless there is new physics not included in standard BBN.

14.60.Pq, 14.60.St, 26.35.+c

Upper limits on the abundance of $^4$He produced in big bang nucleosynthesis (BBN) have been used to limit mixing between active ($\nu_e$, $\nu_\mu$, or $\nu_\tau$) and sterile ($\nu_s$, no standard model interactions) neutrinos [1]. In this paper, we point out and discuss how these constraints are dependant on the adopted primordial deuterium abundance. Previous limits on sterile neutrino mixing have assumed a value for the lower bound on the baryon-to-photon ratio $\eta$ derived from interstellar medium and solar system measurements of deuterium (D) and $^3$He, and models of chemical and galactic evolution. Recent measurements of D/H in quasar absorption systems (QAS) have yielded discordant values of this ratio, some higher than previously derived ranges [2], and some lower [3]. Several factors make an investigation of the primordial D/H dependance of BBN constraints on active-sterile neutrino mixing timely: the discordant QAS measurements of D/H; the fact that future solar neutrino experiments may be able to distinguish and identify $\nu_e - \nu_s$ mixing [4]; and the use of sterile neutrinos in schemes for neutrino masses and mixings that explain all available data [5].

As is well known (*e.g.*, Ref. [6]), the abundance of $^4$He produced by BBN is essentially determined by the ratio of neutron to proton number densities (n/p) at "weak freeze-out" (WFO). WFO occurs when the reactions that interchange neutrons and protons proceed too slowly relative to the expansion rate of the universe to keep n/p at its equilibrium value of n/p $\approx \exp(-\Delta m/T)$. Here $\Delta m \equiv m_n - m_p \approx 1.293$ MeV is the neutron-proton mass difference, and $T$ is the photon temperature. Mixing between active and sterile neutrinos increases $(\mathrm{n/p})_{\mathrm{WFO}}$, and therefore the primordial $^4$He mass fraction $Y$, in two ways. First, active-sterile neutrino mixing effectively brings more degrees of freedom into thermal contact, increasing the energy density and hence the expansion rate of the universe. Second, active-sterile mixing—especially $\nu_e - \nu_s$ mixing—depletes the electron neutrino and antineutrino populations, reducing the rates of the n $\leftrightarrow$ p interconversion reactions. Both of these effects cause n/p to freeze out at a lower temperature.

Using a neutrino ensemble evolution formalism [1,7] that includes both neutrino oscillations (with matter effects) and neutrino collisions, previous authors [1] have produced exclusion plots in the $\delta m^2$-$\sin^2 2\theta$ plane for both $\nu_e - \nu_s$ and $\nu_\mu - \nu_s$ mixing [9]. Here $\delta m^2$

and $\sin^2 2\theta$ are the difference of the squares of the neutrino vacuum mass eigenvalues and a measure of the vacuum mixing angle, respectively, associated with two-flavor neutrino mixing. These studies showed that for $\eta > 2.8 \times 10^{-10}$, both the $\nu_\mu - \nu_s$ solution to the atmospheric neutrino problem [10] and the $\nu_e - \nu_s$ large-angle MSW solution to the solar neutrino problem [11] are excluded for $Y < 0.247$.

In our study of the primordial D/H dependance of BBN constraints on active-sterile neutrino mixing, we have employed the same neutrino evolution formalism [1,7] as previous authors. We have neglected any net lepton number contributed by the neutrinos. (The recently reported effect of active-sterile neutrino mixing *generating* net lepton number does not occur in the regions of parameter space we consider here [8].) In this case, the neutrino and antineutrino sectors evolve identically. A Fermi-Dirac momentum distribution for all neutrinos is assumed, but allowance is made for non-equilibrium number densities. The differential equations in the formalism yield $n_{\nu_e}, n_{\nu_\mu}, n_{\nu_\tau}, n_{\nu_s}$, and $T$ as functions of time. Here $n_{\nu_x}$ denotes the fraction of a full fermionic degree of freedom contributed by neutrino species $x$, which we shall hereafter call the "number density parameter" of neutrino species $x$. In the equations below we will take $n_{\nu_x} = n_{\overline{\nu}_x}$, since we are working under the assumption that the net lepton number contributed by the neutrinos is negligible.

In our BBN computation we have employed the Kawano [12] update of the Wagoner [13] code, with the latest world average neutron lifetime, $\tau = 887.0$ s [14]; the reaction rates of Ref. [15]; and a correction of +0.0031 to $Y$ due to finite nucleon mass and timestep-dependant effects [16]. We have altered the Kawano code to use the 'temperature series' of neutrino number density parameters, $n_{\nu_x}(T)$, to compute the energy density contributed by neutrinos and the n $\leftrightarrow$ p interconversion rates. The neutrino energy density is

$$\rho_\nu = \frac{7}{8}\frac{\pi^2}{15}(n_{\nu_e} + n_{\nu_\mu} + n_{\nu_\tau} + n_{\nu_s})T^4. \tag{1}$$

The n $\leftrightarrow$ p rates are

$$\lambda_{ne\to p\nu} = K\int_1^\infty \left(\frac{1}{1+e^{xz}}\right)\left(1 - \frac{n_{\nu_e}}{1+e^{(x+q)z_\nu}}\right) x(x+q)^2(x^2-1)^{1/2}\,dx, \tag{2}$$

$$\lambda_{n\nu\to pe} = K\int_q^\infty \left(\frac{n_{\nu_e}}{1+e^{(x-q)z_\nu}}\right)\left(\frac{1}{1+e^{-xz}}\right) x(x-q)^2(x^2-1)^{1/2}\,dx, \tag{3}$$

$$\lambda_{n\to pe\nu} = K\int_1^q \left(\frac{1}{1+e^{-xz}}\right)\left(1-\frac{n_{\nu_e}}{1+e^{(q-x)z_\nu}}\right) x(x-q)^2(x^2-1)^{1/2}\,dx, \tag{4}$$

$$\lambda_{pe\to n\nu} = K\int_q^\infty \left(\frac{1}{1+e^{xz}}\right)\left(1-\frac{n_{\nu_e}}{1+e^{(x-q)z_\nu}}\right) x(x-q)^2(x^2-1)^{1/2}\,dx, \tag{5}$$

$$\lambda_{p\nu\to ne} = K\int_1^\infty \left(\frac{n_{\nu_e}}{1+e^{(x+q)z_\nu}}\right)\left(\frac{1}{1+e^{-xz}}\right) x(x+q)^2(x^2-1)^{1/2}\,dx, \tag{6}$$

$$\lambda_{pe\nu\to n} = K\int_1^q \left(\frac{1}{1+e^{xz}}\right)\left(\frac{n_{\nu_e}}{1+e^{(q-x)z_\nu}}\right) x(x-q)^2(x^2-1)^{1/2}\,dx. \tag{7}$$

In these expressions $x \equiv E_e/m_e$, where $E_e$ and $m_e$ are the total electron (or positron) energy and rest mass, respectively; $z \equiv m_e/T$; $z_\nu \equiv m_e/T_\nu$, where $T_\nu$ is the appropriate neutrino temperature; $q \equiv \Delta m/m_e$; and $K$ is a constant obtained by solving the equation

$$(\lambda_{ne\to p\nu} + \lambda_{n\nu\to pe} + \lambda_{n\to pe\nu})\,|_{z\to\infty} = 1/\tau \tag{8}$$

for $K$, where $\tau$ is the experimentally measured neutron lifetime.

The lower limit on $\eta$ obtained from a standard BBN calculation with $N_\nu = 3$ is not appropriate for BBN with active-sterile mixing. This is because the lower limit on $\eta$ depends on the expansion rate, often codified as an effective number of neutrino generations $N_\nu$ [17]. Since active-sterile mixing increases $N_\nu$ (at least for the range of parameter space we consider here [8]), it affects the lower bound on $\eta$. Therefore, we will plot our results as a function of the primordial D/H value—the experimentally determined quantity—rather than as a function of $\eta$. These considerations are most important for the $\nu_\mu - \nu_s$ atmospheric neutrino mixing solution, and much less important (nearly negligible) for the $\nu_e - \nu_s$ small-angle MSW solution to the solar neutrino problem.

In Fig. 1, a representative $\nu_\mu - \nu_s$ atmospheric neutrino mixing solution ($\delta m^2 = 1.0 \times 10^{-2}$ eV$^2$, $\sin^2 2\theta = 0.6$ [1]) is *assumed*, and the resulting BBN $^4$He yield is plotted as a function of the BBN D/H yield. The value of $\eta$ (given as $\eta_{10} = 10^{10}\,\eta$) at various values of D/H is

also indicated on the figure. For a given value of D/H, the implied abundance of $^4$He can be interpreted as the observational upper limit required to constrain the solution. Alternatively, a *detection* of these neutrino mixing parameters by, for example, future atmospheric neutrino experiments would yield an independent determination of the primordial $^4$He abundance, so long as D/H were known from QAS studies. This could be very interesting, given the recent emphasis on the systematic uncertainties in the determination of the primordial $^4$He abundance as derived from helium recombination lines in extragalactic HII regions [18]. The conclusions reached from Fig. 1 are essentially the same over the range of $\delta m^2$ ($10^{-3} - 10^{-1}$ eV$^2$) for the proposed $\nu_\mu - \nu_s$ mixing explanation of the atmospheric neutrino problem.

Fig. 2 is similar to Fig. 1, but with a representative $\nu_e - \nu_s$ small-angle MSW solution to the solar neutrino problem assumed ($\delta m^2 = 4.0 \times 10^{-6}$ eV$^2$, $\sin^2 2\theta = 8.0 \times 10^{-3}$ [19]). These mixing parameters have only a very small effect on the BBN $^4$He yield. For the most recent data from solar neutrino experiments and the standard solar model [20], there is no $\nu_e - \nu_s$ large-angle MSW solution to the solar neutrino problem [19].

Some of the QAS data suggest D/H $\approx 2 \times 10^{-4}$ [2]. Figs. 1-2 show the range D/H $= 1.5 - 2.3 \times 10^{-4}$, as determined in Ref. [21]. This range of D/H implies a lower bound on $\eta$ that is significantly lower than that used in previous studies. Since a lower $\eta$ implies a lower $^4$He yield, high D/H relaxes constraints on any effect that increases the expansion rate, including mixing with sterile species [22]. Fig. 1 shows, however, that for D/H $\approx 2 \times 10^{-4}$, the $\nu_\mu - \nu_s$ atmospheric neutrino solution is still somewhat constrained [23] if current observational inferences [24] of primordial $^4$He are correct: $Y = 0.234 \pm 0.003 \pm 0.005$, where the first error is statistical and the second systematic. Of course, if this $\nu_\mu - \nu_s$ atmospheric neutrino mixing solution were inferred from atmospheric neutrino experiments, and QAS studies confirm D/H $\approx 2 \times 10^{-4}$, the implied $^4$He abundance of $Y \approx 0.245$ would be significantly higher than the central value of $Y = 0.234$ cited above.

The $\nu_e - \nu_s$ small-angle MSW solar neutrino solution is allowed for D/H $\approx 2 \times 10^{-4}$. Fig. 2 shows that an observational upper bound of $Y \lesssim 0.232$ would be required to restrict this small angle solution if such a high D/H is indeed the primordial value.

Other very high quality QAS data—arguably better [3] for the determination of D/H than that used in Ref. [2]—suggest D/H $\approx 2 \times 10^{-5}$ [3]. This value of D/H is incompatible with standard BBN with $N_\nu = 3$ [16,25,17] for current observational inferences of the primordial $^4$He abundances [24], and any mixing with sterile neutrinos would only exacerbate the problem. As mentioned previously, however, it has been argued that $Y$ has been systematically underestimated, and a more appropriate upper limit on $Y$ may actually be $Y \leq 0.255$ [18]. It is unlikely that the systematic error in $Y$ is enough to allow the $\nu_\mu - \nu_s$ atmospheric neutrino solution for D/H $\approx 2\times10^{-5}$. However, observation of the $\nu_e - \nu_s$ small-angle MSW solar neutrino solution, together with a solid determination of D/H $\approx 2\times10^{-5}$, would require that $Y$ has been systematically and significantly underestimated by about 0.015 (see Fig. 2), unless there is non-standard physics during the BBN epoch [25]. This is a somewhat trivial point, since the mixing parameters of the small-angle $\nu_e - \nu_s$ MSW solution produce only very slightly more $^4$He than the standard BBN picture with $N_\nu = 3$, for which the "crisis" at low D/H is well-known [16,25,17]. Useful constraints on the $\nu_e - \nu_s$ small-angle MSW solar neutrino solution would require very precise observational knowledge of $\eta$ and $Y$. This may, however, still be interesting in view of the fact that future solar neutrino experiments may be able to distinguish the sterile neutrino oscillation-based solution from other solutions [4]. Also, many models that seek to satisfy all available constraints on neutrino properties employ the $\nu_e - \nu_s$ small-angle MSW solar neutrino solution [5] (but see Ref. [26]).

## ACKNOWLEDGEMENTS

This work was supported by the National Science Foundation through NSF Grant PHY-9503384, and by a NASA Theory Grant.

## REFERENCES


[1] X. Shi, D. N. Schramm, and B. D. Fields, Phys. Rev. D **48**, 2563 (1993); K. Enqvist, K. Kainulainen, and M. Thomson, Nucl. Phys. B **373**, 498 (1992); and references therein.

[2] M. Rugers and C. J. Hogan, report astro-ph/9603084, 1996 (unpublished); M. Rugers and C. J. Hogan, Astrophys. J. Lett. **459**, L1 (1996); E. J. Wampler *et al.*, Astron. Astrophys. (to be published); R. F. Carswell *et al.*, Mon. Not. R. Astron. Soc. **278**, 506 (1996); A. Songaila, L. L. Cowie, C. J. Hogan, and M. Rugers, Nature **368**, 599 (1994); R. F. Carswell *et al.*, Mon. Not. R. Astron. Soc. **268**, L1 (1994).

[3] S. Burles and D. Tytler, report astro-ph/9603070, 1996 (unpublished); D. Tytler, X. Fan, and S. Burles, report astro-ph/9603069 (unpublished).

[4] P. I. Krastev, S. T. Petcov, and L. Qiuyu, IASSNS - AST 96/11, hep-ph/9602033, 1996 (unpublished); S. M. Bilenky and C. Giunti, Z. Phys. C **68**, 495 (1995); P. I. Krastev and S. T. Petcov, Nucl. Phys. B **449**, 605 (1995). W. Kwong and S. P. Rosen, Phys. Rev. Lett. **68**, 748 (1992).

[5] G. M. Fuller, J. R. Primack, and Y.-Z. Qian, Phys. Rev. D **52**, 1288 (1995); J. J. Gomez-Cadenas and M. C. Gonzales-Garcia, Z. Phys. (to be published); S. Goswami, CUPP-95/4, hep-ph/9507212 (unpublished); E. Ma and P. Roy, Phys. Rev. D **52**, R4780 (1995); E. Ma and J. Pantaleone, Phys. Rev. D **52**, R3763 (1995); R. Foot and R. R. Volkas, Phys. Rev. D **52**, 6595 (1995); Z. G. Berezhianai and R. N. Mohapatra, Phys. Rev. D **52**, 6607 (1995); E. J. Chun, A. S. Joshipura, and A. Y. Smirnov, Phys. Lett. B **357**, 608 (1995).

[6] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Reading, 1990), Chap. 4.

[7] B. H. J. McKellar and M. J. Thomson, Phys. Rev. D **49**, 2710 (1994).

[8] X. Shi, report astro-ph/9602135, 1996 (unpublished); R. Foot, M. J. Thomson, and R.

R. Volkas, UM-P-95/90, 1995 (unpublished).

[9] $\nu_\tau - \nu_s$ mixing would have the same effects on $Y$ as $\nu_\mu - \nu_s$ mixing, but this is of less interest since neither the solar nor atmospheric neutrino anomalies could be solved with $\nu_\tau - \nu_s$ mixing.

[10] T. K. Gaisser, F. Halzen, and T. Stanev, Phys. Rep. **258**, 173 (1995).

[11] W. Haxton, Ann. Rev. Astron. Astrophys. **33**, 459 (1995); J. N. Bahcall, Astrophys. J. (to be published).

[12] L. Kawano, Fermilab-Pub-92/04-A, 1992 (unpublished).

[13] R. V. Wagoner, Astrophys. J. **179**, 343 (1972); R. V. Wagoner, Astrophys. J. Suppl. Ser. **18**, 247 (1969).

[14] Particle Data Group, L. Montanet *et al.*, Phys. Rev. D **50**, 1173 (1994).

[15] M. S. Smith, L. H. Kawano, and R. A. Malaney, Astrophys. J. Suppl. Ser. **85**, 219 (1993).

[16] P. J. Kernan and L. M. Krauss, Phys. Rev. Lett. **72**, 3309 (1994).

[17] C. Y. Cardall and G. M. Fuller, report astro-ph/9603071, 1996 (unpublished).

[18] D. Sasselov and D. Goldwirth, Astrophys. J. Lett. **444**, L5 (1995); C. J. Copi, D. N. Schramm, and M. S. Turner, Science **267**, 192 (1995).

[19] P. I. Krastev and S. T. Petcov in Ref. [4].

[20] J. N. Bahcall and M. Pinsonneault, Rev. Mod. Phys. **64**, 85 (1992).

[21] M. Rugers and C. J. Hogan in Ref. [2].

[22] Recently, this point was noted independently in P. J. Kernan and S. Sarkar, CWRU-P3-96, astro-ph/9603045, 1996 (unpublished).

[23] There is a small but nontrivial discrepancy between our results and the those of Shi *et al.* [1]. For $\eta = 2.8 \times 10^{-10}$, the atmospheric neutrino solution would imply $Y = 0.248$ according to Shi *et al.* [1], while we obtain $Y = 0.252$. The difference is due to our use of the $+0.0031$ correction to $Y$, and the fact that we numerically integrate the altered n$\leftrightarrow$p reactions in our BBN calculation. Thus the constraints on the $\nu_\mu - \nu_s$ atmospheric neutrino solution are not as relaxed as might have been expected for D/H $\approx 2 \times 10^{-4}$.

[24] K. A. Olive and S. T. Scully, Int. J. Mod. Phys. A **11**, 409 (1996).

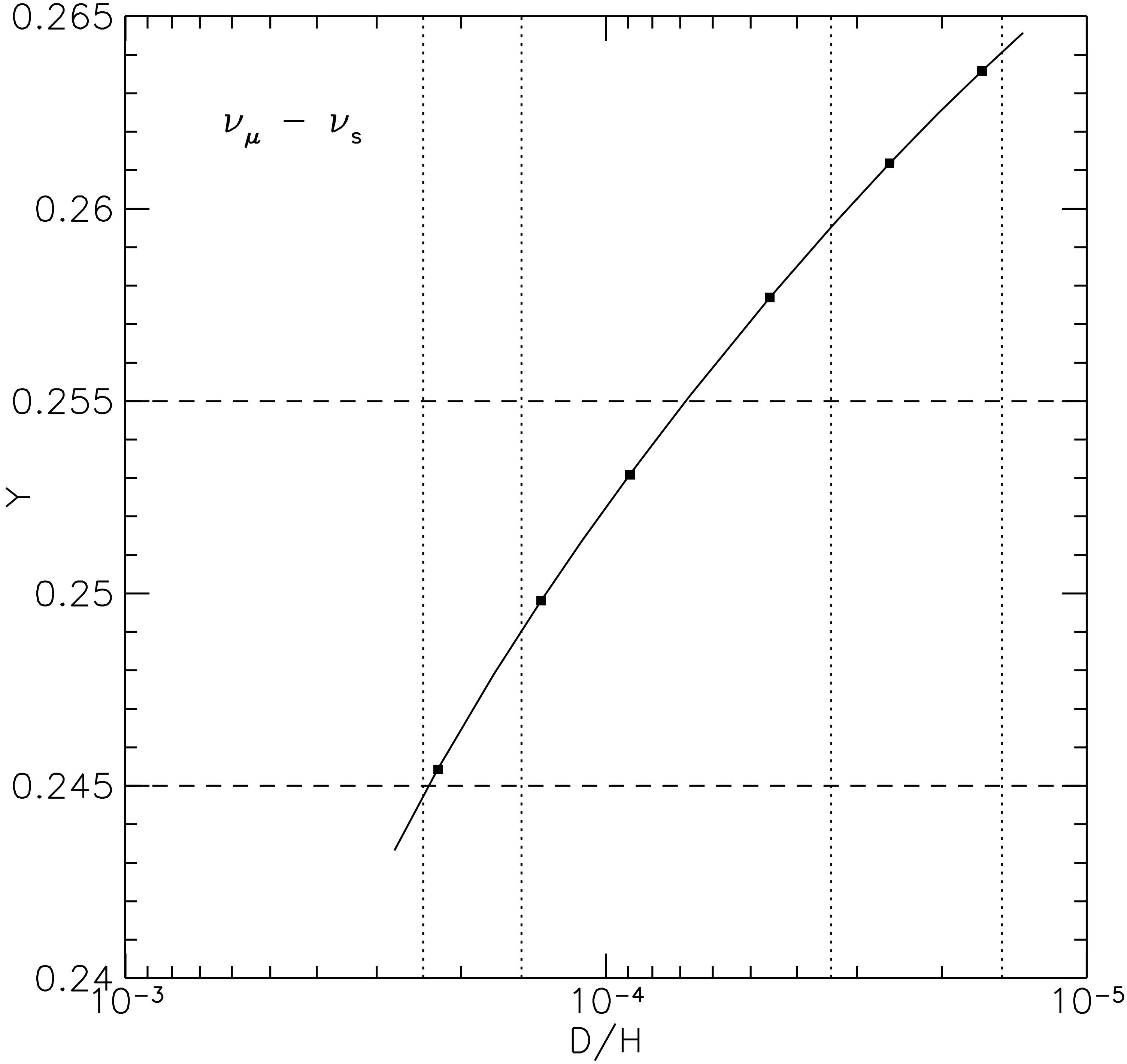
[25] N. Hata et al., Phys. Rev. Lett. **75**, 3977 (1995).

[26] C. Y. Cardall and G. M. Fuller, Phys. Rev. D (to be published 15 Apr. 1996).

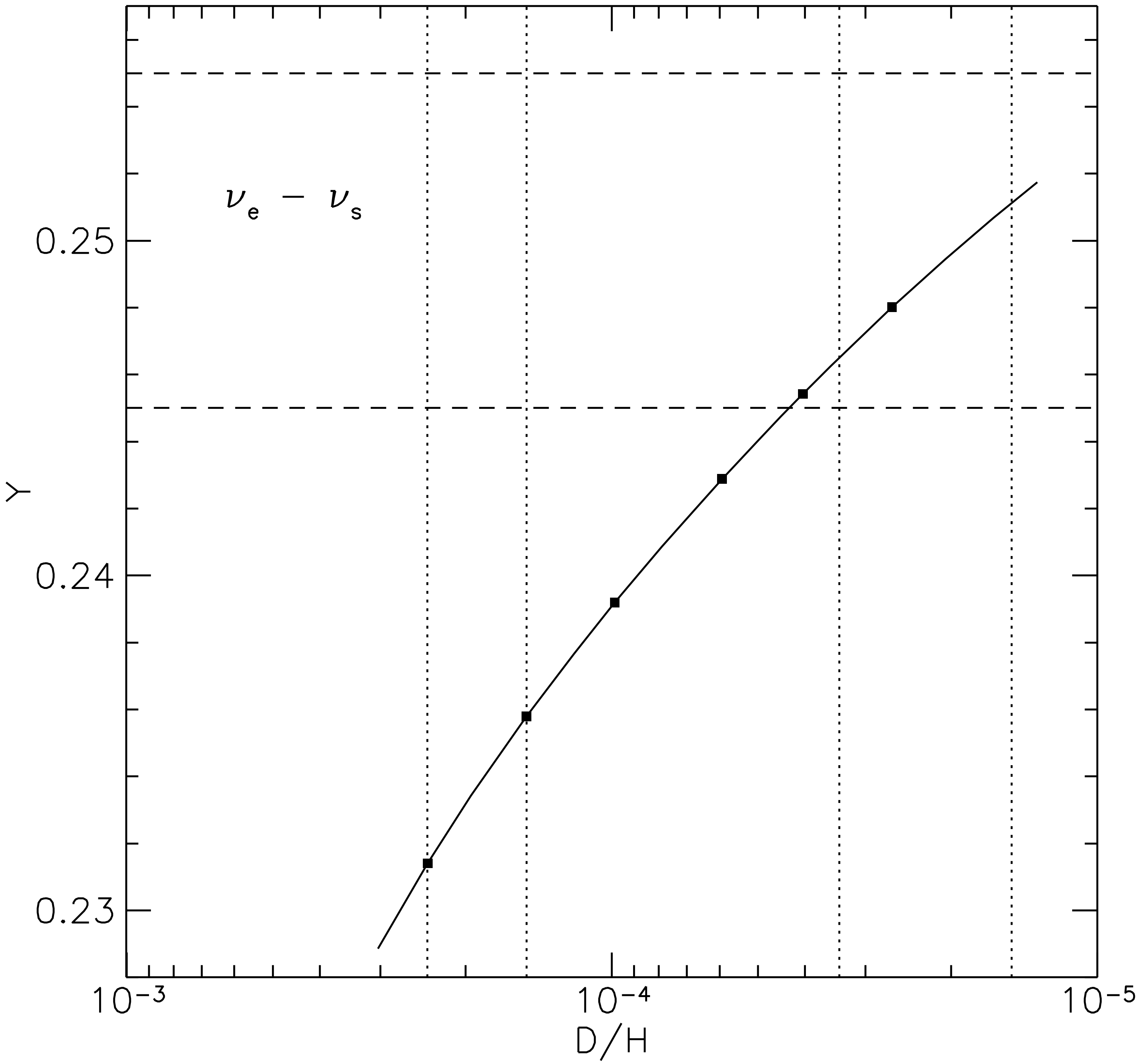
## FIGURES

FIG. 1. BBN yields for a typical $\nu_\mu - \nu_s$ atmospheric neutrino solution ($\delta m^2 = 1.0 \times 10^{-2}$ eV$^2$, $\sin^2 2\theta = 0.6$). The solid curve is the $^4$He mass fraction $Y$ vs. $D/H$. The squares indicate, from lower left to upper right, $10^{10}\ \eta = 1.7,\ 2.3,\ 3.0,\ 4.6,\ 6.6,\ 8.6$. The dotted lines indicate the ranges of "high" and "low" D/H inferred from QAS studies. The dashed lines indicate two possible upper limits on $Y$: the currently popular $Y = 0.245$, and the more conservative $Y = 0.255$.

FIG. 2. BBN yields for a typical $\nu_e - \nu_s$ small-angle MSW solution to the solar neutrino problem ($\delta m^2 = 4.0 \times 10^{-6}$ eV$^2$, $\sin^2 2\theta = 8.0 \times 10^{-3}$). The solid curve is the $^4$He mass fraction $Y$ vs. $D/H$. The squares indicate, from lower left to upper right, $10^{10}\ \eta = 1.5,\ 2.0,\ 2.6,\ 3.6,\ 4.6,\ 5.6$. The dotted lines indicate the ranges of "high" and "low" D/H inferred from QAS studies. The dashed lines indicate two possible upper limits on $Y$: the currently popular $Y = 0.245$, and the more conservative $Y = 0.255$.



0.265
0.26
0.255
0.25
0.245
0.24
Y
$\nu_\mu - \nu_s$
$10^{-3}$
$10^{-4}$
$10^{-5}$
D/H



$\nu_e - \nu_s$
0.25
0.24
0.23
Y
$10^{-3}$
$10^{-4}$
$10^{-5}$
D/H